Foundational Numeracy & Operational Fluency

Name: _____

Reasoning and Analyzing: Demonstrate and apply mental math strategies

Understanding and Solving: Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving

Foundational Numeracy (K-2)	(K)	(1)	(2)	(2+)
Understanding **Counting**	Counts 1:1 (touch and talk)	Counting On, Skip Counting, Choral Counting	Counts forward and backward in sequence (+/- 1, +/- 2, 5, 10)	Selects and uses various counting strategies fluently
Understanding & Communicating **Recognize / Subitize**	Decodes values on 6-sided dice and uses a 10-frame	Correctly reads a D10 or D20 die, 10-frame, calendar, Rekenrek, and 100 Grid	Uses a 100 Grid, Base-10 models, and dice to explore & investigate place value	Writes number stories that include grouping / subitizing
Analyzing **Compare / Order**	Can order values when concrete models are given	Can compare and order values less than 20	Compares and orders values up to 100	Compares values beyond or between other values (2.5 is less than 3, but more than 2)
Reasoning **Compose / Combine**	Making Ten ___ + ___ = 10	+/- 0, 1, 2 Doubles 10 + ____	Near doubles (double +/- 1, 2) Compensation (borrowing)	Manipulates and combines multiple strategies effectively
Representing* **Modeling Numbers**	Encodes/Decodes concrete visual models of numbers (1:1 models)	Connects number & equivalent concrete visual models (matching equivalent values)	Creates number sequences & patterns (increasing and decreasing)	Interprets and extends number patterns and models

*Suggested Resources: Tiny Dots card game, Interactive 100s Maze

KG: Numbers to 10

1: Numbers to 20 (recommend 31 to connect with **calendar and weather**)

2: Numbers to 100 (connecting to money concepts **– focus on whole dollars**

Operational Fluency Trajectory (Grade 3-5)

Operational Fluency (3-5)	(3)	(4)	(5)	
Understanding Additive (+/-) Strategies	Doubles, near doubles, +/- 0, 10 , 100, 1000 Model addition w/ base 10 blocks	Combos of 1, 10, 100, 1000, ex. adding bills and coins (parts and wholes)	Compensation (regroup / borrow) adding and subtract #s up to 6 digits, using place value	Selects and uses various additive strategies fluently
Understanding Multiplicative (x / ÷) Strategies	Multiples of 2, 10, 5 20, 50, 100	Doubling, Multiples of 3, 6, 9, 4	Knows multiples of 7, 8, 11	Squares, Near squares
Reasoning Compares number families	Even, Odd, x10, x5 Compare value of digits according to place value	Identify multiples of 3, 6, 9, 4 Finds patterns in groups of facts	Determines common multiples compares prime and composite #s	Determines common factors
Solving Compose / Decompose / Partitive Reasoning	Understands multiplication as repeated addition	Performs multiplication by counting by groups Decompose number into powers of 10	Uses properties of composite and prime numbers	Decomposes numbers flexibly & fluently in new contexts
Representing Modeling whole numbers* and unit fractions	Encodes/Decodes concrete visual models of numbers	Connects shape & space with number concepts (arrays, perimeter & area)	Creates visual models of whole and fractional numbers (pie and bar models)	Interprets and extends models to include "improper fractions" and mixed numbers

*Recommended Tool: animated prime factorisation visualization (moving dots)

Grade 3: Place value up to 1 000s (2-D models, arrays), benchmark fractions

Grade 4: to 10 000s, modular models (clocks), decimals (hundredths), Canadian currency (all)

Grade 5: to 1 000 000s, decimals (thousandths), elapsed time, metric conversion (ex. mg to g to kg)

Operational Fluency Trajectory (Grade 5-7)

Operational Fluency (5-7)	(5)	(6)	(7)	
Understanding Additive (+/-) Strategies	+/- 0, 1 ,2 Doubles, Making 10, Near Doubles (regroup / borrow) Add and subtract using place value	Selects and uses various additive strategies including standard algorithm	Applies additive strategies to decimals, fractions and mixed numbers	Uses additive strategies to work with decimals, fractions and percents in new contexts
Understanding Multiplicative (x / ÷) Strategies	Multiples of 2, 10, 5 Doubling/Halving, Multiples of 3, 6, 9, 4	Multiples of 7, 8, 11 Applies Multiplication algorithms	Squares, Near squares in multiplication table Uses division algorithm	Flexibly and fluently uses multiplication and division strategies to solve novel problems
Solving Recognize number families	Identify numbers as Even, Odd, x10, x5 multiples of 3, 6, 9, 4	Uses knowledge of multiples to determine common multiples	Applies common multiples to simplify and solve problems involving fractions	Solve various problems involving fractions with unlike denominators
Reasoning Proportions and part: whole relationships	Understands multiplication as grouping	Compares groups using ratios or proportions	Solves problems requiring proportional reasoning	Distinguishes between rates, ratios and proportions and uses each appropriately depending on the context
Representing Modeling Number*	Plots values on a number line	Represents / models equivalent fractions and percents	Locates points within a cartesian plane using coordinates (x,y)	Models proportional relationships using a double number line or a ratio table
Communicating Using words and symbols	Uses > < = ≈ to compare values	Uses correct notation and vocabulary for basic operations, equality, fractions, ratios, and percents	Solutions are detailed and organized, including symbols, words, and notations where needed	Accurately and cleanly communicates reasoning in various contexts. May include inequality (≠), arrows, columns, etc.

Algebraic Reasoning (Symbolic Logic) Trajectory (Gr 7-9)

Reasoning and Analyzing: Use **reasoning and logic** to explore, analyze, and apply mathematical ideas

Understanding and Solving: Apply **multiple strategies** to solve problems in both **abstract** and **contextualized** situations
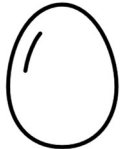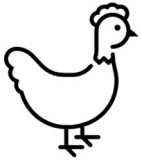
Communicating and Representing
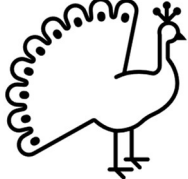
Algebraic Reasoning	(7)	(8)	(9)	(10+)
Understanding	Substitutes values into single variable expressions	Substitutes into and evaluates expressions with more than one variable	Simplifies and evaluates expressions using exponents and polynomials	Converts concrete models into variable expressions and equations
Solving	Solves 1-step and whole number equations independently	Solves 2-step equations independently	Solves multi-step equations and isolates variables in common formulas	Writes and solves equations based on novel contexts or multi-step problems
Reasoning (Logic Strategies)	Relies on strategic guessing	Uses inverse operations and logic strategies	Uses both algebraic and concrete models to solve equations	Manipulates equations and combines multiple strategies effectively
Communicating	Encodes / identifies solution using the form: variable = ___ "Work" is shown but does not use standard notation	Shows / Identifies at least one intermediate step between the problem and the solution	Communicates multiple algebraic steps in a logical and conventional manner	Algebraic notation is complete, organized, and communicates an efficient way to solve problems May include limits, estimates, and/or non-permissible values
Representing Modeling expressions and equations	Encodes/Decodes concrete visual models of expressions	Connects equations and equivalent concrete visual models (Mobiles)	Creates visual models of linear equations (Graphing) $y = ax$ $y = ax + b$	Interprets visual models of equations (intercepts, slope, interpolation, extrapolation)

- Grade 7: two-step equations with **whole-number** coefficients, constants, and solutions
- Grade 8: two-step equations with **integer** coefficients, constants, and solutions
- Grade 9: multi-step one-variable linear equations

Fact Fluency Trajectory (K-9)

Name : _____

Date : _____

Fluency Stage	Addition*	Subtraction	Multiplication	Division (Factoring)
	Count On, 1 : 1, using tools	Count Back From, 1 : 1, using tools	Skip Counting, May use fingers	Guess and revise strategy Ex. Dividing a number by any number smaller than itself, chosen randomly.
	Making Ten	Think of related addition fact	Knows Benchmarks (x1, x10, x5, x2)	Strategic elimination (use a list of prime numbers 2, 3, 5, 7, … and eliminate one at a time)
	Compensation (Regrouping) Ex. 18 + 7 = 18 + (2 + 5) = (18 +2) + 5 = 25	Take from 10 (compensation) Ex. 21-8 =21-10+2 =11+2 =13	Adds a group onto a benchmark multiple* 6 x 9 =5 x 9 +9 = 45+9	Recognizes fact families (remembers patterns in products of 2,5,10)
	Doubles and Near-Doubles	Subtract in parts (partition) Ex. 21-8 =21-1-7	Subtract a group from a benchmark multiple* 8 x 9 =8 x 10 – 8 = 80-8 = 72	Divides in parts (according to place value or benchmarks) 85 ÷ 5 = (40÷5)+ (40÷5)+ (5÷5) = 8 + 8 + 1 = 17
	Rounds one addend up and then uses subtraction 28+17 =28+20-3 =48-3	Same distance = same difference 200-18 = 199-17 (reduce both by 1) 199 - 17 182	Uses squares or other known facts Ex. 7 x 8 = 7 x 7 + 7 = 49 + 7 = 56	Thinks of related multiplication fact (connects to inverse operation) Ex. 60 ÷ 5 = 12 because 12 x 5 = 60

*Benchmark sums: 10s, doubles
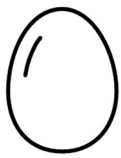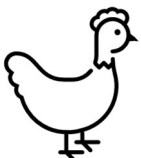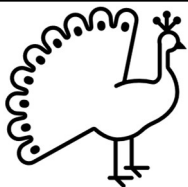
*Benchmark multiples: x 1, 2, 5, 10   then 2, 4, 8   then 3, 6, 9   then 7 , 11 , 12, 13

Fractions/Decimals/Percents Fluency Trajectory

Name : _____

Date : _____

Fluency Stage	Addition	Subtraction	Multiplication	Division
(egg)	Adds fractions with common denominators Adds decimals to one place (no carrying)	Subtracts fractions with common denominators	Multiplies benchmark fractions by a whole number	Divides fractions to simplify (2, 3, 5, 10)
(cracked egg)	Adds fractions up to and over 1, same denominator. Uses a bar or pie models	Subtracts fractions up to and over 1, with same denominator. Uses bar or pie models	Uses multiplication facts to create equivalent fractions Uses array or grid models to represent multiplication	Reduces fractions using strategic elimination (use a list of prime numbers 2, 3, 5, 7, … and eliminate one at a time) Divides a whole number by a decimal value.
(chick)	Adds mixed numbers and decimals by grouping whole and fractional parts	Subtracts mixed numbers and decimals by grouping whole and fractional parts	Uses multiplication to create common denominators when needed	Recognizes and converts decimals and fractions as two ways of showing the same value. (A fraction is a division, and a decimal is a division by a power of 10)
(hen)	Adds fractions with unlike denominators Adds decimals accurately	Subtracts fractions with unlike denominators Subtracts decimals accurately	Multiplies fractions Multiplies decimal values accurately	Divides fractions by a whole number and vice versa Divides a fraction by another fraction
(turkey)	Fluently converts values to fractions, decimals and percents to add them efficiently	Uses subtraction to solve problems including decimals, fractions and percentages (ie. tax, sale price)	Uses a ratio table (proportional reasoning) to solve rate and ratio problems.	Uses and solves proportions (equates 2 fractions to find an unknown value)

Dear teachers,

This set of foundational numeracy trajectories may be helpful for the purposes of assessment and reporting. It includes a multigrade scope and sequence of number concepts and skills. Feel free to copy and edit as needed.

The column entitled "Proficient" describes proficiency at the upper end of the grade band that is included in the table. For example, in the Foundational Numeracy Trajectory (K-2), the "Proficient" column describes the targets for the **end of grade 2**. A student in grade 1 who is already demonstrating that level of proficiency would be considered extending for grade 1.

There are 6 tables:

- K-2 (Foundational Numeracy)
- 3-5 (Intermediate Years, Operational Fluency)
- 5-7 (Middle Years, Number Concepts)
- 7-9 (Algebra concepts)
- Fact Fluency (K-9)
- Fractions/Decimals/% (5+)

How could teachers use this resource?

- [ ] Selecting one row or column to include as a mini rubric on a quiz or assignment
- [ ] Building a set of lessons or a review guide, based on the previous grade-level descriptors
- [ ] Identifying a student's current level of proficiency (especially when below or above current grade level)
- [ ] Using the next row or column to inform next steps for instruction and areas of growth (helpful for writing comments)
- [ ] Use the proficiency table to identify and develop instructional materials that are below/at/above grade-level (helps with differentiation)